Exam. Code : 211002
Subject Code : 4900

M.Sc. (Mathematics) 2nd Semester (Batch 2021-23)

MATH-564 : MECHANICS—II

Time Allowed—3 Hours] [Maximum Marks—100

Note :— Attempt *five* questions in all, selecting at least *one* question from each section. The fifth question may be attempted from any section. All questions carry equal marks.

SECTION—A

1. (a) Show that the rate of change of linear momentum for a system of particles is equal to the vector sum of the external forces acting on the system.

   (b) Show that the angular momentum of the system about a fixed point O is equal to that about O of a single particle of total mass equal to that of the entire system, concentrated at its centroid and moving with the centroid velocity, together with the angular momentum about the centroid of the system in its motion relative to the centroid.

2. (a) A uniform circular disc of mass M and radius a is rotating in its plane with initial angular velocity w, its centre being at rest. If a point on the rim be suddenly fixed, find the new angular velocity of the disc and the velocity of its centre.

(b) A uniform rod AB of mass M and length 2a lies at rest on a smooth horizontal table. An impulse J is applied at A in the plane of the table and perpendicular to the rod. Determine the velocity of the centroid and the angular velocity of the rod.

SECTION—B

3. Derive Euler's dynamical equations for the motion of a rigid body about a fixed point.

4. (a) Show that the motion of a rigid body about a fixed may be represented by the rolling of an ellipsoid fixed in the body upon a plane fixed in space.

   (b) A uniform solid sphere rolls without slipping on a rough horizontal plane which is rotating with uniform angular velocity about a vertical axis. If there are no forces acting on the sphere save its weight and the friction at the contact, prove that the locus of the centre of the sphere is a circle.

SECTION—C

5. (a) Derive Lagrange's equations of motion for a holonomic dynamical system.

   (b) Using Lagrange's equations, obtain the equations of motion of a planet describing a central orbit.

6. (a) Derive kinetic energy as a quadratic function of generalized velocities.

(b) The ends of uniform rod AB, of length 2a cos 15° and weight W are constrained to slide on a smooth circular wire of radius a, fixed with its plane vertical. The end A is connected by an elastic string, of natural length a and modulus $\frac{W}{2}$, to the highest point of the wire. If θ is the angle which is the perpendicular bisector of the rod makes with the downward vertical, show that the potential energy V is given by :

$$V = -\frac{Wa}{2}\left[\cos(\theta - 75^\circ) + 2\cos\frac{\theta}{2}(\theta + 75^\circ)\right] + \text{constant}$$

SECTION—D

7. (a) Derive Euler-Lagrange's equations.

(b) Find the extremals of the functional

$$\int_0^1 \left[2x + \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2\right] dt$$ such that $x(0) = 1$, $y(0) = 1$, $x(1) = 1.5$, $y(1) = 1$.

8. (a) State and prove principle of Least Action.

(b) Solve the boundary value problem

$$y'' + y = -\frac{x^3}{5}, \quad y(0) = 0,\ y(1) = 1$$

by Rayleigh-Ritz method.